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A Computational Scheme for Structural Influence Coefficients of Certain Planar Wings

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The objective of this work has been to reduce the amount of numerical computation work involved in obtaining structural influence coefficients, especially for special-purpose computer programs involving wings that have elastic axes that can be approximated by a series of straight line noncollinear segments. The need for repetitive runs, probably due to different mass arrangements, enhances the desirability of the technique. A general matrix of influence coefficients which allows the user to handle the N-segmented case is derived and presented. The decreased number of integral evaluations is summarized for a three-segmented case as an illustration [only 12 integrals compared to 27 (straight); only 12 integrals compared to 42 (noncollinear)]. For collinear segments of practical wings, the order of required integral evaluations is shown to differ by N^2 and by more for noncollinear cases. Therefore, a significant reduction in computation effort is claimed.

Nomenclature

C_{ij}^{AB} = influence coefficient (deflection at the ith point in the A sense due to a load at j in the B sense)
D_{ik}^{A} = portion of influence coefficient (intermediate result)
EI = bending stiffness in aircraft coordinate system
$\overline{EI}$ = bending stiffness in local coordinate system
f = frequency
GJ = torsional rigidity in aircraft coordinate system
$\overline{GJ}$ = torsional rigidity in local coordinate system
M_α = pitch moment due to unit load, or unit roll moment, or unit pitching moment
M_θ = roll moment due to unit load, or unit roll moment, or unit pitching moment
$\overline{m}_k$ = moment at kth point (referred to local coordinate system) due to unit force, etc.
N,n = number of wing segments
P_z = unit vertical load
${}^iS_R^k$ = integral portions of C_{ik}^{AB} [see Eqs. (18, 21, and 28) for definitions]
T_i = torque at ith point
X, Y, Z = aircraft coordinate system
$\bar{X}, \bar{Y}, \bar{Z}$ = local coordinate system
α = pitch angle designator
Δ_{ik}^{A} = displacement of the A sense at the kth point due to a load at k with a rigid element between the ith point and the kth point
δ_{ik}^{A} = displacement of the A sense at the ith point due to load at k
ξ_{ik}, η_{ik} = distances referred to X, Y, respectively, between the ith and kth points
θ = roll angle designator
Λ_k = the angle between the axis Y and the kth segment

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Introduction

WHILE current interests in finite element techniques have certainly lessened the use of other methods of solving realistic structures, there are reasons for alternate approaches. Among them are 1) possible computational advantages such as repetition and numerical stability and 2) checkout uses such as cases when there is a wide gap between classical (simplified) solutions and the realistic structure of interest. There is at least one current text[1] that extols the merits and simplicity of virtual work and approaches related to influence coefficients. Other experimental methods of deriving structural influence coefficients have been addressed in recent literature.[2,3] The authors are well aware of the capabilities of NASTRAN and other general finite element modeling.[4] The advantages of such programs and techniques are aptly demonstrated in their use by the authors and in the current literature. On the other hand, there can be numerical difficulties, and this paper describes an alternative method of computing the cantilever influence coefficients in a global coordinate system for a wing modeled by a series of connected straight beams that are coplanar, but not collinear. The method offers an improved computational efficiency in evaluating the integral formulas[5,6] associated with classical beam equations, derived by applying Castigliano's theorem or virtual work[7] which are, in simplest form, given by

$$C_{ij}^{ZZ} = \int_0^\ell \frac{m_i m_j}{EI} \mathrm{d}\eta \tag{1}$$

$$C_{ij}^{\alpha\alpha} = \int_0^\ell \frac{T_i T_j}{GJ} \mathrm{d}\eta \tag{2}$$

Throughout the development of this method, the properties of integrals and the principle of superposition are employed to make use of previously computed expressions, thus significantly reducing the overall number of computations and numerical evaluations of the integral formulas compared to more classical methods. Alternate means of obtaining influence coefficients are of interest,[8-10] especially if some special advantage results such as joining subcomponents together.

The method is simply a manipulation of Eqs. (1) and (2), derived by Castigliano's theorem, to obtain forms that are easily computed and simply automated for use by hand or on a digital computer. The equations obtained are of closed form (closed in the sense that only a different form of the equations are obtained) using only one assumption beyond simple beam theory. This very realistic assumption, that sweep angle is less than 90 deg, is used in formulating the moment distributions. Three degrees of freedom, vertical translation Z, pitch angle α, and roll angle θ are used in the development of the structural influence coefficients. These lead to the following matrix of influence coefficients:

$$[C] = \begin{bmatrix} C^{ZZ} & C^{Z\theta} & C^{Z\alpha} \\ C^{\theta Z} & C^{\theta\theta} & C^{\theta\alpha} \\ C^{\alpha Z} & C^{\alpha\theta} & C^{\alpha\alpha} \end{bmatrix} \tag{3}$$

Wing Modeled with Segmented Beam

Consider a general wing elastic axis as shown in Fig. 1. The elastic axis is approximated by a series of straight line segments. The procedure is to examine the diagonal elements of the subcomponents of the influence coefficients, then to obtain the superdiagonal and subdiagonal terms by the Maxwell and Betti-Rayleigh reciprocal theorems. The purpose is to arrive at a simple relationship using previously computed elements for as many of the matrix terms as possible. For convenience, spanwise coordinates Y_i are arranged in monotonically increasing order.

The notation for the distances between two points in the Y and X directions, respectively, is introduced as

$$\eta_{ij} = Y_j - Y_i \tag{4}$$

$$\xi_{ij} = X_j - X_i \tag{5}$$

For the special case when the unit load is at the point where the displacement is desired (i.e., $k=j$), the method is to compute the deflection at the kth point as if the beam were rigid between i and k (k being the next point past i; i.e., $k=i+1$) using information already computed (or the end conditions when $k=1$), then add the computed displacement of the beam as if it were cantilevered at the ith point. The stepwise procedure is as follows:

1) Resolve the unit load at Y_k back to the Y_i point such that the resolved loads give the same moment distribution up to Y_i.
2) Using the resolved loads, compute the displacement at the ith point using the influence coefficients already computed.
3) Compute the displacement of the k^{th} point, using small angle theory, as if the beam were rigid between Y_i and Y_k.
4) Evaluate the displacement due to the flexibility between Y_i and Y_k.
5) Superimpose the two displacements at Y_k by adding the results of steps 3 and 4.

As an example of the five steps mentioned above, a unit load P_Z is placed at (X_k, Y_k) and the influence coefficients C^{ZZ}_{kk}, $C^{\theta Z}_{kk}$, $C^{\alpha Z}_{kk}$ are computed in terms of C^{AB}_{ii} (or the end conditions if $k=1$). The equivalent loads at (X_i, Y_i) due to a unit load P_Z at (X_k, Y_k) are

$$P_Z \; = 1 \tag{6}$$

$$M_\theta \; = \eta_{ik} \tag{7}$$

$$M_\alpha \; = \xi_{ik} \tag{8}$$

The displacements at (X_i, Y_i) due to the equivalent loads are

$$\delta^Z_{ik} = P_Z C^{ZZ}_{ii} + M_\theta C^{Z\theta}_{ii} + M_\alpha C^{Z\alpha}_{ii} \tag{9}$$

$$\delta^\theta_{ik} = P_Z C^{\theta Z}_{ii} + M_\theta C^{\theta\theta}_{ii} + M_\alpha C^{\theta\alpha}_{ii} \tag{10}$$

$$\delta^\alpha_{ik} = P_Z C^{\alpha Z}_{ii} + M_\theta C^{\alpha\theta}_{ii} + M_\alpha C^{\alpha\alpha}_{ii} \tag{11}$$

Extrapolating from the ith point to the kth point using small angle theory, the displacements at (X_k, Y_k) without the flexibility between i and k are

$$\Delta^Z_{ik} = \delta^Z_{ik} + \eta_{ik}\delta^\theta_{ik} + \xi\delta^\alpha_{ik} \tag{12}$$

$$\Delta^\theta_{ik} = \delta^\theta_{ik} \tag{13}$$

$$\Delta^\alpha_{ik} = \delta^\alpha_{ik} \tag{14}$$

The displacement due only to the flexibility between the Y_i and Y_k points is obtained by using the basic simple beam equations presented in the Introduction. These equations are in an elastic axis system which must be denoted as such, i.e., the bar system. Equation 15 is used to evaluate the Z direction displacement due to a unit load in the Z direction

$$D^Z_{ik} = \int (\bar{m}^2_k / \overline{EI})\,\mathrm{d}\bar{\eta} \tag{15}$$

Note that transformations for values in the elastic axis system to the aircraft axis system are given by

$$\bar{m}_k = m_k / \cos\Lambda_k \tag{16}$$

$$\bar{\eta} \; = \eta / \cos\Lambda_k \tag{17}$$

Since the only flexibility of interest is between Y_i and Y_k, the integration is only over this range. Thus Eq. (15) becomes

$$D^Z_{ik} = \frac{1}{\cos^3\Lambda_k} \int_{Y_i}^{Y_k} \frac{(Y_k - \eta)^2}{\overline{EI}}\,\mathrm{d}\eta \tag{18}$$

or by replacing the integral with definition ${}^iS^k_1$, Eq. (15) becomes

$$D^Z_{ik} = (1/\cos^3\Lambda_k)\,{}^iS^k_1 \tag{19}$$

Using a similar approach, still for the unit load P_Z, the integrals associated with rolling and pitching motions become

$$D^\theta_{ik} = (1/\cos\Lambda_k)\,{}^iS^k_2 \tag{20}$$

$$D^\alpha_{ik} = (\sin^2\Lambda_k / \cos\Lambda_k)\,{}^iS^k_4 \tag{21}$$

where

$${}^iS^k_2 = \int_{Y_i}^{Y_k} \frac{Y_k - \eta}{\overline{EI}}\,\mathrm{d}\eta$$

and

$${}^iS^k_4 = \int_{Y_i}^{Y_j} (1/\overline{GJ})\,\mathrm{d}\eta$$

Superimposing the solution gives the influence coefficient. Equations (12) and (19) give the deflection in the Z direction:

$$C^{ZZ}_{kk} = \Delta^Z_{ik} + D^Z_{ik} \tag{22}$$

Using Eqs. (9-12), (18) and (22), the following is obtained

$$C^{ZZ}_{kk}=C^{ZZ}_{ii}+\eta_{ik}C^{Z\theta}_{ii}+\xi_{ik}C^{Z\alpha}_{ii}+\eta_{ik}C^{\theta Z}_{ii}+\eta^2_{ik}C^{\theta\theta}_{ii}$$
$$+\eta_{ik}\xi_{ik}C^{\theta\alpha}_{ii}+\xi_{ik}C^{\alpha Z}_{ii}+\xi_{ik}\eta_{ik}C^{\alpha\theta}_{ii}+\xi^2_{ik}C^{\alpha\alpha}_{ii}$$
$$+(1/\cos^3\Lambda_k)\,{}^iS^k_1 \tag{23}$$

For the roll angle due to a unit vertical load, Eqs. (13) and (20) are added to obtain

$$C^{\theta Z}_{kk}=C^{\theta Z}_{ii}+\eta_{ik}C^{\theta\theta}_{ii}+\xi_{ik}C^{\theta\alpha}_{ii}+\frac{1}{\cos\Lambda_k}\,{}^iS^k_2 \tag{24}$$

and for the pitch angle, Eqs. (14) and (21) are added to obtain

$$C^{\alpha Z}_{kk}=C^{\alpha Z}_{ii}+\eta_{ik}C^{\alpha\theta}_{ii}+\xi_{ik}C^{\alpha\alpha}_{ii}+\frac{\sin^2\Lambda_k}{\cos\Lambda_k}\,{}^iS^k_2 \tag{25}$$

Using the same five-step procedure as above, only with a unit rolling moment and torque, the following are obtained:

$$C^{\theta\theta}_{kk}=C^{\theta\theta}_{ii}+(\cos\Lambda_k)\,{}^iS^k_3+\frac{\sin^2\Lambda_k}{\cos\Lambda_k}\,{}^iS^k_4 \tag{26}$$

$$C^{\alpha\theta}_{kk}=C^{\alpha\theta}_{ii}+({}^iS^k_3-{}^iS^k_4)\sin\Lambda_k \tag{27}$$

$$C^{\alpha\alpha}_{kk}=C^{\alpha\alpha}_{ii}+\frac{\sin^2\Lambda_k}{\cos\Lambda_k}\,{}^iS^k_3+{}^iS^k_4\cos\Lambda_k \tag{28}$$

where

$${}^iS^k_3=\int_{Y_i}^{Y_j}(1/\overline{EI})\,d\eta$$

By the Betti-Rayleigh reciprocal theorem, it is noted that

$$C^{Z\theta}_{kk} = C^{\theta Z}_{kk} \tag{29}$$

$$C^{\theta\alpha}_{kk} = C^{\alpha\theta}_{kk} \tag{30}$$

$$C^{Z\alpha}_{kk} = C^{\alpha Z}_{kk} \tag{31}$$

For the special case when the unit load is outboard of the point where the displacement is desired (i.e., $j>i$ when $Y_1<Y_2<Y_3<\cdots<Y_N$), the method is to compute the deflection of the ith point using loads that when resolved to the point located at Y_i, give the same load distribution inboard of that point as if the unit load were located at Y_j. The stepwise procedure is as follows:

1) Resolve the unit load at Y_j back to Y_i such that the same load distribution results inboard of the Y_i position.

2) Using the resolved loads, compute the displacement at the ith point using the influence coefficients already computed.

As an example of this procedure, a unit load P_Z is placed at (X_j, Y_j), and the influence coefficients C^{ZZ}_{ij}, $C^{\theta Z}_{ij}$, and $C^{\alpha Z}_{ij}$ are computed in terms of C^{AB}_{ii}. The equivalent loads at (X_i, Y_i) due to a unit load P_Z at (X_j, Y_j) are

$$P_Z = 1 \tag{32}$$

$$M_\theta = \eta_{ij} \tag{33}$$

$$M_\alpha = \xi_{ij} \tag{34}$$

The displacements at (X_i, Y_i) due to these equivalent loads are

$$\delta^Z_{ij} = P_ZC^{ZZ}_{ii}+M_\theta C^{Z\theta}_{ii}+M_\alpha C^{Z\alpha}_{ii} \tag{35}$$

$$\delta^\theta_{ij} = P_ZC^{\theta Z}_{ii}+M_\theta C^{\theta\theta}_{ii}+M_\alpha C^{\theta\alpha}_{ii} \tag{36}$$

$$\delta^\alpha_{ij} = P_ZC^{\alpha Z}_{ii}+M_\theta C^{\alpha\theta}_{ii}+M_\alpha C^{\alpha\alpha}_{ii} \tag{37}$$

When the values of P_Z, M_θ, and M_α are substituted into Eqs. (35-37), the following three influence coefficients of interest are obtained:

$$C^{ZZ}_{ij} = C^{ZZ}_{ii}+\eta_{ij}C^{Z\theta}_{ii}+\xi_{ij}C^{Z\alpha}_{ii} \tag{38}$$

$$C^{\theta Z}_{ij} = C^{\theta Z}_{ii}+\eta_{ij}C^{\theta\theta}_{ii}+\xi_{ij}C^{\theta\alpha}_{ii} \tag{39}$$

$$C^{\alpha Z}_{ij} = C^{\alpha Z}_{ii}+\eta_{ij}C^{\alpha\theta}_{ii}+\xi_{ij}C^{\alpha\alpha}_{ii} \tag{40}$$

Using the same two-step procedure as above, only with a unit rolling moment and torque (with respect to the aircraft axis system), the following are obtained:

$$C^{Z\theta}_{ij} = C^{Z\theta}_{ii} \tag{41}$$

$$C^{\theta\theta}_{ij} = C^{\theta\theta}_{ii} \tag{42}$$

$$C^{\alpha\theta}_{ij} = C^{\alpha\theta}_{ii} \tag{43}$$

$$C^{Z\alpha}_{ij} = C^{Z\alpha}_{ii} \tag{44}$$

$$C^{\theta\alpha}_{ij} = C^{\theta\alpha}_{ii} \tag{45}$$

$$C^{\alpha\alpha}_{ij} = C^{\alpha\alpha}_{ii} \tag{46}$$

Using the Maxwell and Betti-Rayleigh reciprocal theorems, the subdiagonals can be computed by the relationship

$$C^{BA}_{ji}=C^{AB}_{ij} \tag{47}$$

Hence the equations to compute the matrix of influence coefficients are shown in Eqs. (23-31) and (38-47). The small angle assumption used in computing the diagonal terms is consistent with the original formulation of the beam equations. Use of the monotonically increasing values of Y_i is solely for convenience in presentation. The assumption here is unimportant as long as the moment distribution is correctly computed. In examining Eqs. (23-31), it is noted that several equations are not defined for $|\Lambda_i|=90$ deg. Also, the moment distributions given by Eqs. (7,8) and similar equations, are inadequate for $|\Lambda_i|\geq 90$ deg. Thus, a restriction of $|\Lambda_i|<90$ deg must be made in using the results of this paper. Of course, the use of monotonically increasing values of Y_i will ensure that this restriction is met.

A summary of the influence coefficients [Eqs. (23-31) and (38-47)] is shown in Table 1, consistent with Eq. (3) and the aforementioned definitions of the integral terms denoted by S. By examination of this table, it is seen that the kth diagonal elements are a function of the ith diagonal elements with additional terms from the flexibility between the ith and kth points. The off-diagonal terms are a function of only the diagonal terms.

It should be noted that Table 1 has not included effects such as 1) shear deformation; 2) yaw, fore and aft, and compression degrees of freedom; 3) nonrigid joints, and 4) noncoplanar wings. If these are present, the formulation must be changed or the effects superimposed as required.

Examples

As brief examples of the technique, consider the top view of the wing/beam model shown in Fig. 2. For $\Lambda_1=\Lambda_2=\Lambda_3=0$, a direct Castigliano approach requires 27 integral evaluations in order to determine the 9×9 matrix of influence coefficients. In contrast, the method of this paper requires only 12.

For $\Lambda_2,\Lambda_3\neq 0$, the case shown in Fig. 2, the direct approach requires 42 integral evaluations if EI and GJ are constant for each interval, considerably more if not. The present method again requires only 12 integrals even for EI and GJ non-

Table 1 Summary of influence coefficients—segmented beam

Term	Restriction	Computational expression
Submatrix C^{ZZ}		
$C^{ZZ}_{1\,1}$		${}^{0}S^{1}_{1}/\cos^{3}\Lambda_{1}$
$C^{ZZ}_{k\,k}$	$k\neq 1$ $k=\mathrm{i}+1$	$C^{ZZ}_{i\,i}+\eta_{ik}C^{Z\theta}_{i\,i}+\xi_{ik}C^{Z\alpha}_{i\,i}$ $+\eta_{ik}C^{\theta Z}_{i\,i}+\eta^{2}_{ik}C^{\theta\theta}_{i\,i}+\eta_{ik}\xi_{ik}C^{\theta\alpha}_{i\,i}$ $+\xi_{ik}C^{\alpha Z}_{i\,i}+\xi_{ik}\eta_{ik}C^{\alpha\theta}_{i\,i}+\xi^{2}_{ik}C^{\alpha\alpha}_{i\,i}$ $+{}^{i}S^{k}_{1}/\cos^{3}\Lambda$
$C^{ZZ}_{i\,j}$	$i<j$	$C^{ZZ}_{i\,i}+\eta_{ij}C^{Z\theta}_{i\,i}+\xi_{ij}C^{Z\alpha}_{i\,i}$
$C^{ZZ}_{i\,j}$	$i>j$	$C^{ZZ}_{j\,j}+\eta_{ji}C^{Z\theta}_{j\,j}+\xi_{ji}C^{Z\alpha}_{j\,i}$
[a]Submatrix $C^{Z\theta}$		
$C^{Z\theta}_{1\,1}$		${}^{0}S^{1}_{2}/\cos\Lambda_{1}$
$C^{Z\theta}_{k\,k}$	$k\neq 1$ $k=i+1$	$C^{\theta Z}_{i\,i}+\eta_{ik}C^{\theta\theta}_{i\,i}+\xi_{ik}C^{\theta\alpha}_{i\,i}+{}^{0}S^{1}_{2}/\cos\Lambda_{k}$
$C^{Z\theta}_{i\,j}$	$i<j$	$C^{Z\theta}_{i\,i}$
$C^{Z\theta}_{i\,j}$	$i>j$	$C^{Z\theta}_{j\,j}+\eta_{ji}C^{\theta\theta}_{j\,j}+\xi_{ji}C^{\theta\alpha}_{j\,j}$
[b]Submatrix $C^{Z\alpha}$		
$C^{Z\alpha}_{1\,1}$		${}^{0}S^{1}_{2}\sin^{2}\Lambda_{1}/\cos\Lambda_{1}$
$C^{Z\alpha}_{k\,k}$	$k\neq 1$ $k=i+1$	$C^{Z\alpha}_{i\,i}+\eta_{ik}C^{\theta\alpha}_{i\,i}+\xi_{ik}C^{\alpha\alpha}_{i\,i}$ $+{}^{i}S^{k}_{2}\sin^{2}\Lambda_{k}/\cos\Lambda_{k}$
$C^{Z\alpha}_{i\,j}$	$i<j$	$C^{Z\alpha}_{i\,i}$
$C^{Z\alpha}_{i\,j}$	$i>j$	$C^{Z\alpha}_{j\,j}+\eta_{ji}C^{\theta\alpha}_{j\,j}+\xi_{ji}C^{\alpha\alpha}_{j\,j}$
Submatrix $C^{\theta\theta}$		
$C^{\theta\theta}_{1\,1}$		${}^{0}S^{1}_{3}\cos\Lambda_{1}+{}^{0}S^{1}_{4}\sin^{2}\Lambda_{1}/\cos\Lambda_{1}$
$C^{\theta\theta}_{k\,k}$	$k\neq 1$ $k=i+1$	$C^{\theta\theta}_{i\,i}+{}^{i}S^{k}_{3}\cos\Lambda_{k}+{}^{i}S^{k}_{4}\sin^{2}\Lambda_{k}/\cos\Lambda_{k}$
$C^{\theta\theta}_{i\,j}$	$i<j$	$C^{\theta\theta}_{i\,i}$
$C^{\theta\theta}_{i\,j}$	$i>j$	$C^{\theta\theta}_{j\,j}$
[c]Submatrix $C^{\theta\alpha}$		
$C^{\theta\alpha}_{1\,1}$		$({}^{0}S^{1}_{3}-{}^{0}S^{1}_{4})\sin\Lambda_{k}$
$C^{\theta\alpha}_{kk}$	$k\neq 1$ $k=i+1$	$C^{\theta\theta}_{i\,i}+({}^{i}S^{k}_{3}-{}^{i}S^{k}_{4})\sin\Lambda_{k}$
$C^{\theta\alpha}_{i\,j}$	$i<j$	$C^{\theta\alpha}_{i\,i}$
$C^{\theta\alpha}_{i\,j}$	$i>j$	$C^{\theta\alpha}_{j\,j}$
Submatrix $C^{\alpha\alpha}$		
$C^{\alpha\alpha}_{1\,1}$		${}^{0}S^{1}_{4}\cos\Lambda_{1}+{}^{0}S^{1}_{3}\sin^{2}\Lambda_{1}/\cos\Lambda_{1}$
$C^{\alpha\alpha}_{k\,k}$	$k\neq 1$ $k=i+1$	$C^{\alpha\alpha}_{i\,i}+{}^{i}S^{k}_{4}\cos\Lambda_{k}+{}^{i}S^{k}_{3}\sin^{2}\Lambda_{k}/\cos\Lambda_{k}$
$C^{\alpha\alpha}_{i\,j}$	$i<j$	$C^{\alpha\alpha}_{i\,i}$
$C^{\alpha\alpha}_{i\,j}$	$i>j$	$C^{\alpha\alpha}_{j\,j}$

[a] $C^{\theta Z}_{j\,i}=C^{Z\theta}_{i\,j}$. [b] $C^{\alpha Z}_{j\,i}=C^{Z\alpha}_{i\,j}$. [c] $C^{\alpha\theta}_{j\,i}=C^{\theta\alpha}_{i\,j}$.

constant. In each case, all of the other influence coefficients can be computed from the augmented integrals of symmetry.

For a practical example of this method, it has been applied to a flexible fuselage, folding wing, pylon-carrying aircraft. Both fuselage and wing were modeled using Table 1. The folding wing joint was not considered rigid but was modeled with a pitch and roll spring that was superimposed on the wing influence coefficients. Pylons were coupled to the wing. Excellent agreement was achieved compared to experimental results, as shown in Fig. 3. The first five natural frequencies were closely approximated with the method of this paper, while a straight elastic axis only gave limited acceptable results.

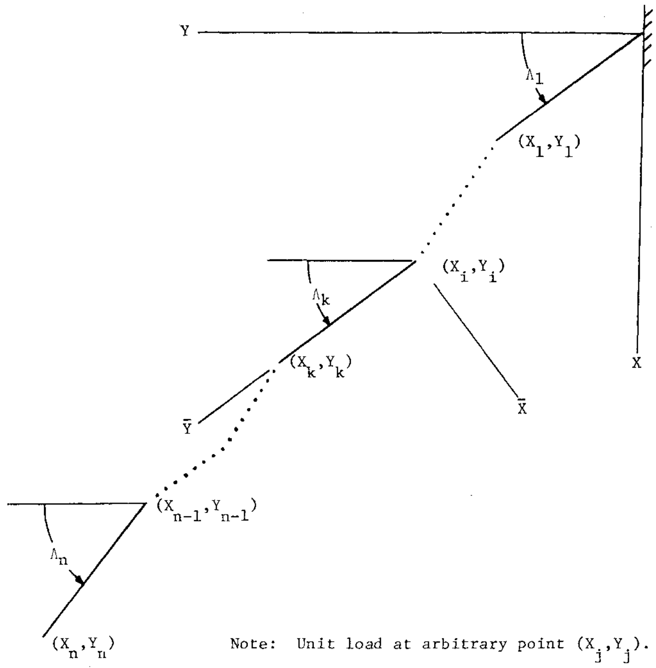

Fig. 1 Wing elastic axes for segmented approach.

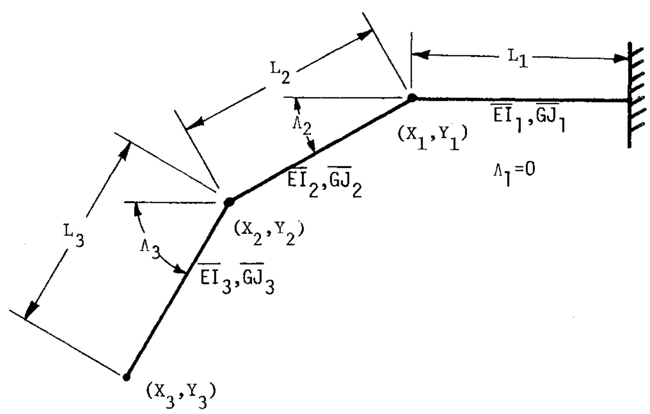

Fig. 2 Illustrative example—three-segment case.

Table 2 Comparison of number of integral relationships required for influence coefficient computations—straight beams

Number of integral relationships	Technique	Remarks
$4N$	This work	No basic restrictions
$5N^2$	Castigliano definition	No use of symmetry or constant stiffness
$\frac{3N(N+1)}{2}+N^2$	Castigliano definition	Use of symmetry and constant stiffness
$\frac{N(N+1)(10N+17)}{6}$	Castigliano definition	Use of symmetry and nonconstant stiffness; every advantage taken in evaluations

Observations and Conclusions

The examples given present evidence of the computational advantages of the technique presented. Two of these examples utilize a three-segment wing, and one is that of a current aircraft. For the case of an arbitrary number of N segments,

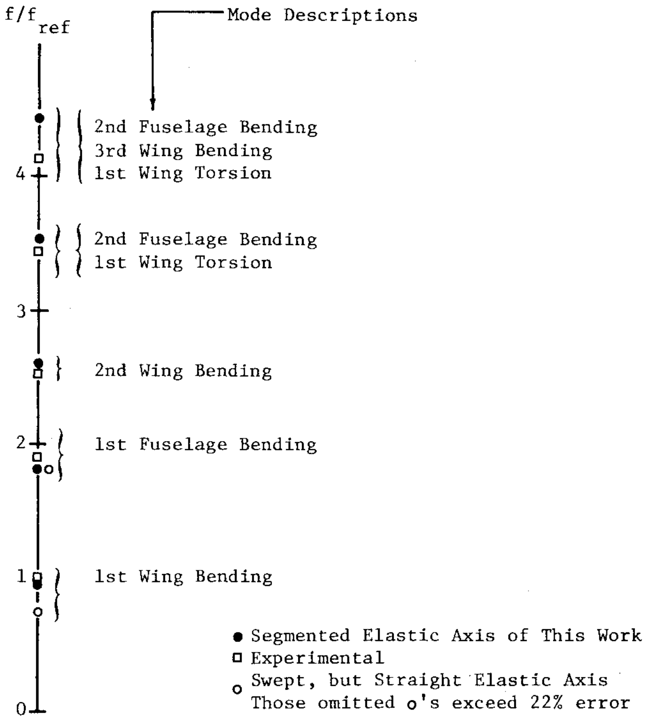

Fig. 3 Comparison with natural frequencies of a current aircraft (f_{ref} = experimental frequency of first wing bending).

general relationships for the required number of integral evaluations exist, at least for straight beams. These are shown in Table 2. A distinct advantage is apparent for the technique of this work. While no such table is presented for beams that are not straight, the advantage for the technique of this paper would be even greater due to the additional coupling terms present.

In summary, when beam theory is used to model an aircraft wing or other component parts, the numerical evaluation of the influence coefficients can be significantly reduced in difficulty by using the techniques presented in this paper. This reduction is more significant for those wings that have an elastic axis which can best be approximated by noncollinear straight line segments.

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